Difference Cordial of Operational Graph Related to Cycle

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Abstract *—Let G be a (p, q) graph. A bijective vertex labeling function $f : V(G) \to \{1, 2, \dots p\}$ is called a difference cordial labeling if for each edge uv, assign the label $|f(u) - f(v)|$ then $|e_f(0) - e_f(1)| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph. In this paper, we prove that cycle with one chord, cycle with twin chords and cycle with triangle admit difference cordial labeling.*

Keywords -Difference cordial, Cycle with one chord, Cycle with twin chord, cycle with triangle, Swastik graph.

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I. INTRODUCTION

Here we consider finite, undirected and simple graph. Let $G = (V, E)$ be a (p, q) graph. For graph theoretical terminology and notations we follow Harary[3].
Ponraj et al.[5] introduced difference cordial labeling and proved that path, cycle, complete graph, complete bipartite graph, star, helm are difference cordial graphs. In [6] the same authors have discussed difference cordial labeling behaviour of triangular snake, quadrilateral snake, double triangular snake, double quadrilateral snake and alternate snakes.

II. MAIN RESULTS

Definition II.1. *A chord of a cycle C_n is an edge joining two non-adjacent vertices of cycle C_n.*

Theorem II.1. *Cycle C_n with one chord is difference cordial graph, where chord forms a triangle with two edges of C_n.*

Proof. Let G be the cycle C_n with one chord. Let $V(G) = \{v_1, v_2, \dots, v_n\}$ and $E(G) = \{v_i v_{i+1}/1 \leq i \leq n-1\} \cup \{v_n v_1\} \cup \{v_2 v_n\}$. Here $|V(G)| = n$ and $|E(G)| = n + 1$. To define vertex labeling function $f : V(G) \to \{1, 2, \dots, n\}$, we consider the following cases.

Case 1: n is odd.
$f(v_i) = i; 1 \leq i \leq 2.$

Subcase 1: $n \equiv 1(mod 4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \leq i \leq \frac{n-1}{4}. \\ (2n+1) - 4i; & \frac{n+3}{4} \leq i \leq \frac{n-1}{2}. \end{cases}$$

$$f(v_{2i+2}) = \begin{cases} 4i+1; & 1 \leq i \leq \frac{n-1}{4}. \\ 2n - 4i; & \frac{n+3}{4} \leq i \leq \frac{n-3}{2}. \end{cases}$$

Subcase 2: $n \equiv 3(mod 4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \leq i \leq \frac{n-3}{4}. \\ (2n+1) - 4i; & \frac{n+1}{4} \leq i \leq \frac{n-1}{2}. \end{cases}$$

$$f(v_{2i+2}) = \begin{cases} 4i+1; & 1 \leq i \leq \frac{n-3}{4}. \\ 2n - 4i; & \frac{n+1}{4} \leq i \leq \frac{n-3}{2}. \end{cases}$$

Case 2: n is even.
$f(v_i) = i; 1 \leq i \leq 2.$

Subcase 1: $n \equiv 0(mod 4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \leq i \leq \frac{n}{4}. \\ (2n+2) - 4i; & \frac{n+4}{4} \leq i \leq \frac{n-2}{2}. \end{cases}$$

$$f(v_{2i+2}) = \begin{cases} 4i+1; & 1 \leq i \leq \frac{n-4}{4}. \\ 2n - 4i; & \frac{n}{4} \leq i \leq \frac{n-2}{2}. \end{cases}$$

Subcase 2: $n \equiv 2(mod 4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \leq i \leq \frac{n-2}{4}. \\ (2n+1) - 4i; & \frac{n+2}{4} \leq i \leq \frac{n-2}{2}. \end{cases}$$

$$f(v_{2i+2}) = \begin{cases} 4i+1; & 1 \le i \le \frac{n-2}{4}. \\ 2n-4i; & \frac{n+2}{4} \le i \le \frac{n-2}{2}. \end{cases}$$

In each case cycle C_n with one chord satisfies condition for difference cordial labeling. □

Illustration II.1. *Difference cordial labeling of C_{11} with one chord is shown in Figure 1.*

Definition II.2. *Two chords of a cycle C_n are said to be twin chords if they form a triangle with an edge of the cycle C_n.*
For positive integer n and p with $3 \le p \le n-2$, $C_{n,p}$ is the graph consisting of a cycle C_n with a pair of twin chords with which the edges of C_n form cycles C_p, C_3 and C_{n+1-p} without chords.

Theorem II.2. *Cycle with twin chords $C_{n,3}$ is difference cordial.*

Proof. Let $|V(C_{n,3})| = \{v_1, v_2, \dots, v_n\}$ and $|E(C_{n,3})| = \{v_iv_{i+1}/1 \le i \le n-1\} \cup \{v_nv_1\} \cup \{v_2v_n\} \cup \{v_3v_n\}$. $|V(C_{n,3})| = n$, $|E(C_{n,3})| = n+2$. To define vertex labeling function $f : V(C_{n,3}) \to \{1,2,\dots,n\}$, we consider the following cases.
Case 1: n is odd.
Subcase 1: $n \equiv 1(mod4)$

$$f(v_{2i-1}) = \begin{cases} 4i-3; & 1 \le i \le \frac{n+3}{4}. \\ (2n+5)-4i; & \frac{n+7}{4} \le i \le \frac{n+1}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-2; & 1 \le i \le \frac{n-1}{4}. \\ (2n+2)-4i; & \frac{n+3}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

Subcase 2: $n \equiv 3(mod4)$

$$f(v_{2i-1}) = \begin{cases} 4i-3; & 1 \le i \le \frac{n+1}{4}. \\ (2n+5)-4i; & \frac{n+5}{4} \le i \le \frac{n+1}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-2; & 1 \le i \le \frac{n+1}{4}. \\ (2n+2)-4i; & \frac{n+5}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

Case 2: n is even.
Subcase 1: $n \equiv 0(mod4)$

$$f(v_{2i-1}) = \begin{cases} 4i-3; & 1 \le i \le \frac{n}{4}. \\ (2n+4)-4i; & \frac{n+4}{4} \le i \le \frac{n}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-2; & 1 \le i \le \frac{n}{4}. \\ 2n-4i; & \frac{n}{4} \le i \le \frac{n-2}{2}. \end{cases}$$

Subcase 2: $n \equiv 2(mod4)$

$$f(v_{2i-1}) = \begin{cases} 4i-3; & 1 \le i \le \frac{n+2}{4}. \\ (2n+4)-4i; & \frac{n+6}{4} \le i \le \frac{n}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-2; & 1 \le i \le \frac{n+2}{4}. \\ (2n+3)-4i; & \frac{n+6}{4} \le i \le \frac{n}{2}. \end{cases}$$

In each case cycle with twin chords $C_{n,3}$ satisfies condition for difference cordial labeling. □

Illustration II.2. *Difference cordial labeling of cycle C_{16} with twin chords is shown in Figure 2.*

Definition II.3. *A cycle with triangle is a cycle with three chords which by themselves form a triangle.*
For positive integers p, q, r and $n \ge 6$ with $p+q+r+3 = n$, $C_n(p,q,r)$ denotes a cycle with triangle whose edges form the edges of cycles $C_{p+2}, C_{q+2}, C_{r+2}$ without chords.

Theorem II.3. *Cycle with triangle $C_n(1,1,n-5)$ is difference cordial.*

Proof. Let $|V(C_n(1,1,n-5))| = \{v_1, v_2, \dots, v_n\}$ and $|E(C_n(1,1,n-5))| = \{v_iv_{i+1}/1 \le i \le n-1\} \cup \{v_nv_1\} \cup \{v_1v_3\} \cup \{v_3v_{n-1}\} \cup \{v_{n-1}v_1\}$. $|V(C_n(1,1,n-5))| = n$, $|E(C_n(1,1,n-5))| = n+3$. To define vertex labeling function $f : V(C_n(1,1,n-5)) \to \{1,2,\dots,n\}$, we consider the following cases.
Case 1: n is odd.
$f(v_1) = 1$.
Subcase 1: $n \equiv 1(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \le i \le \frac{n-1}{4}. \\ 2n-4i; & \frac{n+3}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-1; & 1 \le i \le \frac{n-1}{4}. \\ (2n+3)-4i; & \frac{n+3}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

Subcase 2: $n \equiv 3(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \le i \le \frac{n-3}{4}. \\ 2n-4i; & \frac{n+1}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-1; & 1 \le i \le \frac{n+1}{4}. \\ (2n+3)-4i; & \frac{n+5}{4} \le i \le \frac{n-1}{2}. \end{cases}$$

Case 2: n is even.
$f(v_1) = 1$.
Subcase 1: $n \equiv 0(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \le i \le \frac{n}{4}. \\ (2n+1)-4i; & \frac{n+4}{4} \le i \le \frac{n-2}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-1; & 1 \le i \le \frac{n}{4}. \\ (2n+2)-4i; & \frac{n+4}{4} \le i \le \frac{n}{2}. \end{cases}$$

Subcase 2: $n \equiv 2(mod4)$

$$f(v_{2i+1}) = \begin{cases} 4i; & 1 \le i \le \frac{n-2}{4}. \\ (2n+1)-4i; & \frac{n+2}{4} \le i \le \frac{n-2}{2}. \end{cases}$$

$$f(v_{2i}) = \begin{cases} 4i-1; & 1 \le i \le \frac{n-2}{4}. \\ (2n+2)-4i; & \frac{n+2}{4} \le i \le \frac{n}{2}. \end{cases}$$

In each case the cycle with triangle $C_n(1,1,n-5)$ satisfies the condition for difference cordial labeling. □

Illustration II.3. *Difference cordial labeling of cycle C_{11} with triangle ($C_{11}(1,1,6)$) is shown in Figure 3.*

Definition II.4. *[4] Swastik graph is a union of four copies of C_{4n} ($n \in \mathbb{N}-1$). Consider $v_{k,i}$ ($1 \le k \le 4, 1 \le i \le 4n$) to be the vertices of k^{th} copy of C_{4n}, where $v_{k,4n} = v_{k+1,1}, 1 \le k \le 3$ and $v_{4,4n} = v_{1,1}$. It is denoted as Sw_n, $n \in \mathbb{N}-1$.*

Theorem II.4. *Swastik Sw_n is difference cordial graph, $n \in \mathbb{N} - 1$.*

Proof. Let $v_{k,i}$ $(1 \leq k \leq 4, 1 \leq i \leq 4n)$ be the vertices of k^{th} copy of C_{4n} in swastik graph Sw_n, where $v_{k,4n} = v_{k+1,1}, 1 \leq k \leq 3$ and $v_{4,4n} = v_{1,1}$. Here $|V(Sw_n)| = 16n - 4$ and $|E(Sw_n)| = 16n$. We define vertex labeling function $f : V(G) \rightarrow \{1, 2, \ldots, |V(Sw_n)|\}$ as follows.

$$f(v_{1,(2i-1)}) = \begin{cases} 4i - 3; & 1 \leq i \leq n. \\ (8n+3) - 4i; & n+1 \leq i \leq 2n. \end{cases}$$

$$f(v_{1,(2i)}) = \begin{cases} 4i - 2; & 1 \leq i \leq n. \\ 8n - 4i; & n+1 \leq i \leq 2n-1. \end{cases}$$

$$f(v_{2,(2i-1)}) = \begin{cases} 4(n+i-1); & 1 \leq i \leq n. \\ 2(6n - 2j + 1); & n+1 \leq i \leq 2n. \end{cases}$$

$$f(v_{2,(2i)}) = \begin{cases} 4(n+i) - 3; & 1 \leq i \leq n. \\ 4(3n - i) - 1; & n+1 \leq i \leq 2n-1. \end{cases}$$

$$f(v_{3,(2i-1)}) = \begin{cases} 4(2n+i) - 5; & 1 \leq i \leq n. \\ 4(4n - i) + 1; & n+1 \leq i \leq 2n. \end{cases}$$

$$f(v_{3,(2i)}) = \begin{cases} 4(2n+i-1); & 1 \leq i \leq n. \\ 2(8n - 2j - 1); & n+1 \leq i \leq 2n-1. \end{cases}$$

$$f(v_{4,(2i-1)}) = \begin{cases} 11n + 4i - 2; & 1 \leq i \leq n. \\ (19n - 4) - 4i; & n+1 \leq i \leq 2n. \end{cases}$$

$$f(v_{4,(2i)}) = \begin{cases} 4(3n+i) - 5; & 1 \leq i \leq n. \\ (19n+1) - 4i; & n+1 \leq i \leq 2n-1. \end{cases}$$

□

Illustration II.4. *Difference cordial labeling of Sw_4 is shown in Figure 4.*

III. FIGURES

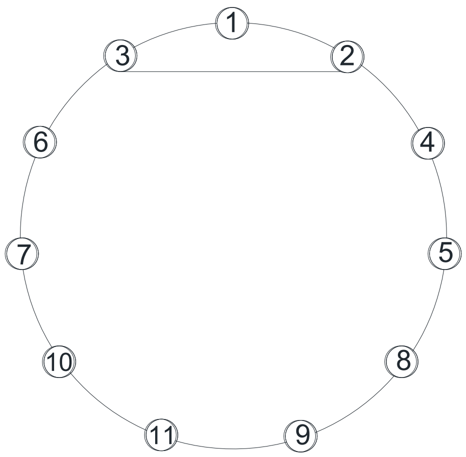

Fig. 1

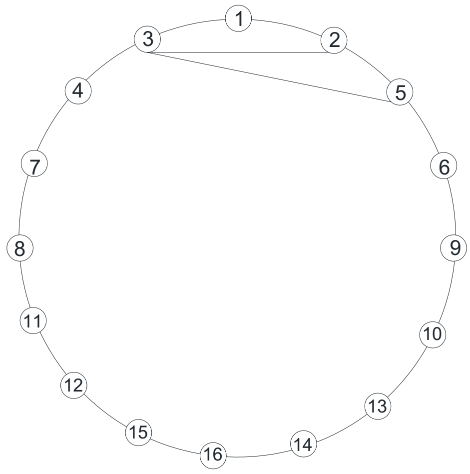

Fig. 2

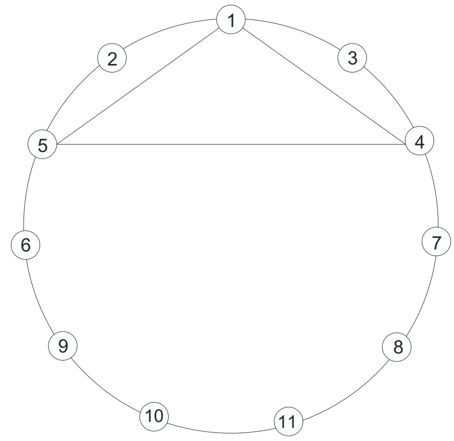

Fig. 3

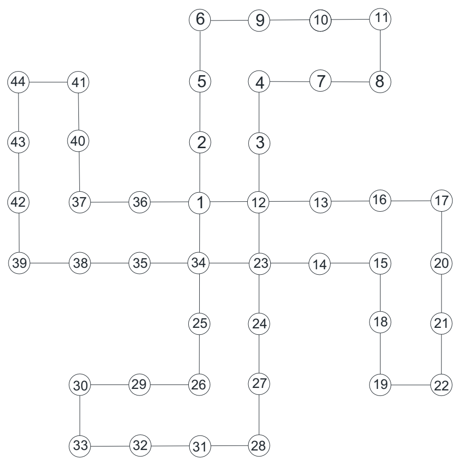

Fig. 4

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IV. CONCLUSION

Difference cordial labeling for different cycle graphs have been discussed. As all cycle graphs are difference cordial by adding one or more chords in cycle C_n. We proved that the resultant graphs are also difference cordial.

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